POLİTEKNİK DERGİSİ

JOURNAL of POLYTECHNIC

ISSN: 1302-0900 (PRINT), ISSN: 2147-9429 (ONLINE)

URL: http://dergipark.org.tr/politeknik

Curve fitting initial guess for iterative differential quadrature solution of burgers equation

Burgers denkleminin iterarif diferansiyel quadrature çözümü için eğri uydurmalı başlangıç tahmini

***(Authors)**: Zekeriya GİRGİN [1], Faruk Emre AYSAL [2], Hüseyin BAYRAKÇEKEN [3]*

ORCID[1]: 0000-0001-5958-9735

ORCID[2]: 0000-0002-9514-1425

ORCID[3]: 0000-0002-1572-4859

Bu makaleye şu şekilde atıfta bulunabilirsiniz(To cite to this article): Girgin Z., Aysal F. E. and Bayrakçeken H., "Curve fitting initial guess for iterative differential quadrature solution of burgers equation", ***Journal Of Polytechnic***, 25(2): 699-709, (2022).

Erişim linki (To link to this article): http://dergipark.org.tr/politeknik/archive

DOI: 10.2339/politeknik.821806

Curve Fitting Initial Guess for Iterative Differential Quadrature Solution of Burgers Equation

Highlights

- *Burgers Equation is solved getting dt=0.01 by using I-DQM, firstly in the literature.*
- *Mean Absolute Error (MAE) of the I-DQM solution of is obtained acceptable*
- *Burgers Equation was solved for Kinematic Viscosity Values υ=0.0001*
- *Firstly, in the literature Curve Fitting Initial Guess is used for iteration.*

Graphical Abstract

The solution of Burgers Equation (BE) performed for dt=0.001 and dt=0.0001 commonly in the literature. In this study, numerical solution of BE carried out by using Iterative Differential Quadrature Method (I-DQM), as dt=0.01. Numerical solutions are obtained even for Kinematic Viscosity Values υ=0.0001.

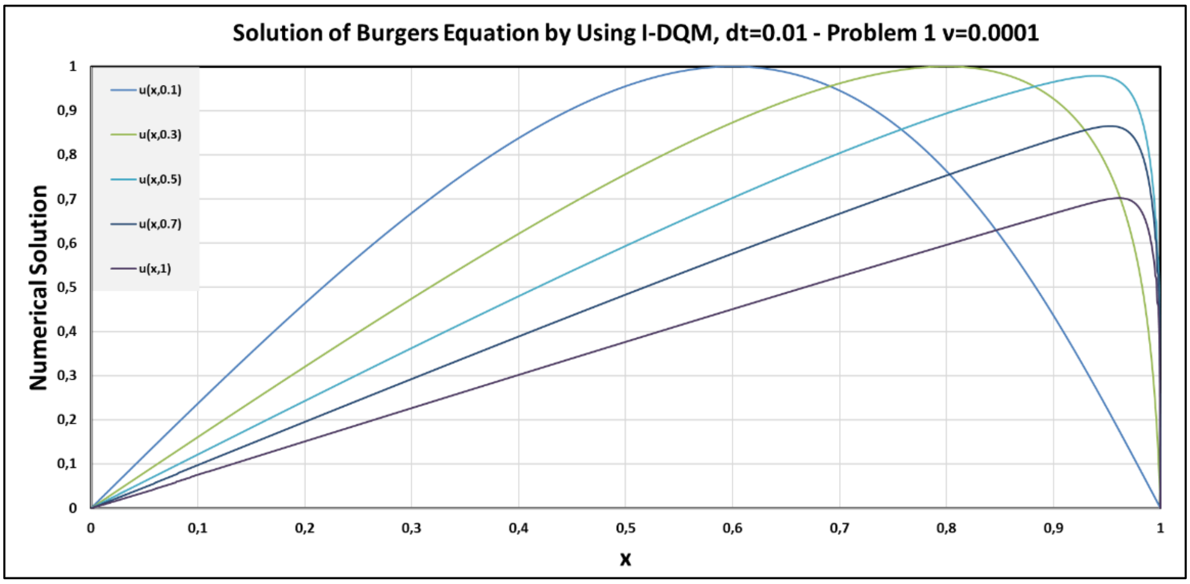

Figure. Numerical Solution of Burgers Equation by Using I-DQM, dt=0.01 - Problem 1 ν=0.0001

Aim

According to presented numerical studies in the literature, the solution of Burgers Equation (BE) performed for dt=0.001 and dt=0.0001 commonly. The aim of this study is the reduce to computation time by means of step-size on time-wise of the solution procedure.

Design & Methodology

Numerical solution of BE is performed by using I-DQM. Iteration procedure of the numerical scheme accomplished by using Newton-Raphson Iteration Method.

Originality

Curve Fitting Initial Guess proposed for initial prediction of iteration. Thereby, time-wise step-size is considered as dt=0.01 which is bigger value for apprehensive DQM solutions of BE in the literature.

Findings

The MAE of the I-DQM solutions Problem 1 and Problem 2 are obtained as $1.04x10^{-4}$ and $9.3x10^{-5}$, respectively.

Conclusion

I-DQM is ensured very high accuracy for solution BE. Besides, step size of time wise is used as dt=0.01 in the proposed method. Thus, an accurate solution is reached in a shorter time than previous studies.

Declaration of Ethical Standards

The author(s) of this article declare that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission.

Curve Fitting Initial Guess for Iterative Differential Quadrature Solution of Burgers Equation

Araştırma Makalesi / Research Article

Zekeriya GİRGİN[1], Faruk Emre AYSAL[2*], Hüseyin BAYRAKÇEKEN[2]

[1] Mühendislik Fakültesi, Makine Müh. Bölümü, Pamukkale Üniversitesi, Türkiye

[2] Teknoloji Fakültesi, Otomotiv Mühendisliği Bölümü, Afyon Kocatepe Üniversitesi, Türkiye

(Geliş/Received : 05.11.2020 ; Kabul/Accepted : 27.12.2020 ; Erken Görünüm/Early View : 30.01.2021)

ABSTRACT

According to presented numerical studies in the literature, the solution of Burgers Equation (BE) performed for dt=0.001 and dt=0.0001 commonly. In this study, numerical solution of BE carried out by using the Iterative Differential Quadrature Method (I-DQM), as dt=0.01. Convergence speed and accuracy of iterative methods depends on the initial guess. Every Partial Differential Equation (PDE) describes one or more than one physical problems from the perspective of the engineering view. Unlike the previous iterative studies, in this work, an initial guess value is used in accordance with the physical nature of the discussed problem by using curve fitting. Absolute error analysis of obtained results performed for comparison with some previous studies. The consequence of comparisons shows that “acceptable results and faster solution could be obtained by using I-DQM with curve fitting initial guess.

Keywords: Burgers equation, iterative differential quadrature method, initial guess, newton raphson method.

Bugers Denkleminin İterarif Diferansiyel Quadrature Çözümü için Eğri Uydurmalı Başlangıç Tahmini

ÖZ

Literatürde sunulan sayısal çalışmalara göre, Burgers Denkleminin (BE) çözümü yaygın olarak dt = 0.001 ve dt = 0.0001 için yapılmıştır. Bu çalışmada BE'nin sayısal çözümü, İteratif Diferansiyel Quadrature Yöntemi (I-DQM) kullanılarak dt = 0.01 olarak gerçekleştirilmiştir. İteratif yöntemlerin yakınsama hızı ve doğruluğu başlangıç tahmini değerine bağlıdır. Her Kısmi Diferansiyel Denklem (PDE), mühendislik bakış açısından bir veya daha fazla fiziksel problemi tanımlar. Önceki iteratif çalışmalardan farklı olarak, bu çalışmada, eğri uydurma kullanılarak tartışılan problemin fiziksel doğasına uygun bir başlangıç tahmini değeri kullanılmıştır. Elde edilen sonuçların mutlak hata analizi, önceki bazı çalışmalarla karşılaştırılmak üzere yapılmıştır. Karşılaştırmaların sonucu, eğri uydurmalı başlangıç tahmini ile I-DQM kullanılarak uygun hassasiyette doğru sonuçların ve hızlı bir çözümün elde edilebileceğini göstermektedir.

Anahtar Kelimeler: Burgers denklemi, iteratif diferansiyel quadrature metodu, başlangıç tahmini, newton raphson metodu.

1. INTRODUCTION

BE is a homogeneous time-dependent one dimensional quasi-linear parabolic PDE. Steady state solution of BE was provided by Bateman in 1915 firstly [1]. In 1948, J. M. Burgers was used BE, given Eq. 1, as mathematical model of turbulent flow. Boundary and Initial conditions of BE is given in Eq. 2 and 3, respectively.

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial x^2} - u\frac{\partial u}{\partial x}$$
$$(x,t) \in \Omega\, x\, (0,T]\ ;\ \Omega = (0,1)\ ;\ \upsilon > 0 \tag{1}$$

$$u(x,0) = f(x), \qquad 0 \le x \le 1 \tag{2}$$

$$u(0,t) = f_1(t), \qquad u(1,t) = f_2(t), 0 \le t \le T \tag{3}$$

Boundary and initial conditions of BE is given in Eq. 2 and 3. Here, “υ” and “u” describe the kinematic viscosity of obtaining flow and the velocity of flow in x direction respectively. Nonlinear convection term and the diffusion term of BE are derived from the kinematic viscosity. These terms show that BE could be considered as simplified form of Navier-Stokes Equations. Besides, BE was used as the mathematical model of the many different problems such as the turbulent flow, gas dynamics, traffic flows, shock wave theory and continuous stochastic processes [2, 3].

Analytic solution of BE could be obtained by using some approximations like Hopf-Cole Transformation [4, 5]. Owing to this situation provide an opportunity for researchers focused on the developing new nonlinear numerical solution methods, the studies about BE increased in recent years [6-10]. Kutluay et al., in 1999, performed analytical solution of BE. Furthermore, numerical solution of BE was solved by using Finite Difference Method (FDM) [10]. Each analytical and numerical solutions were provided the reliable accuracy for kinematic viscosity value of the flow υ=0.01. Kutluay et al. (2004) modified Finite Elements Method (FEM) by

**Sorumlu Yazar (Corresponding Author)*
e-posta : faysa@aku.edu.tr

using Quadratic B-Spline and was solved BE for υ=0.001 [11]. Mittal and Jiwari (2012) tried to solve BE by using Polynomial DQM (PDQM). Nonlinear PDEs was converted to a system of nonlinear Ordinary Differential Equation (ODE) by using PDQM. This ODE system was solved numerically by using Forth Order Runge-Kutta Method. The obtained results were shown that accuracy of this approximation was worse than other solution in the literature. However, the studies have done proved that the PDQM could be easily applied to nonlinear problems [15]. Gupta and Ray (2014) solved Boussinesq-BE comparatively by using Homotopy Perturbation Method (HPM) and Optimum Homotopy Asymptotic Method (OHAM). According to comparison OHAM was provided more sufficient solution than HPM for high order nonlinear fluids mechanic problems [16]. Nascimento et al. performed a comparative analysis for numerical solution of BE by using Fourier Pseudospectral Method (FPM) and Finite Volume Method (FVM). As a result, FPM was ensured an accurate solution but FVM was provided faster solution than FPM. Also, a new hybrid method was developed by combining of FPM and Immersed Boundary Method (IMD). This method ensured the proper solution for BE [17]. Jiwari introduced a hybrid method for solution of BE by using Uniform Haar Wavelet, Quasi-linearization and Implicit Euler Method. Comparing with analytical solution of Hopf-Cole's this hybrid algorithm was provided an appropriate accuracy even for very small kinematic viscosity values, as υ=0.001 [18, 19]. Tamsir et al. were modified the DQM's weighting coefficients by using exponentially modified cubic B-spline function as test function. This new method was called as Expo-MCB-DQM. One and two dimensional BE was solved via Expo-MCB-DQM. Results were shown in this new technique are adequately proper for solution of nonlinear problems [20]. Girgin et al. performed numerical solution of BE by using I-DQM even for very small kinematic viscosity values with high accuracy by using dt=0.001 [21].

According to literature, the solution procedure of BE considered as dt=0.001 or dt=0.0001 for ensuring reliable results in many studies [10-21]. Besides, many effective methods such as DSC, FEM, and HDQ have been used in the literature to solve such as some mechanical and fluid problems [15-24]. In this study, numerical solution of BE is performed by using I-DQM. Reducing the time-wise step-size is decreased to accuracy of iterative methods. However, initial guess of iteration could be changed the accuracy of solution procedure. So, if initial guess would be chosen sufficiently suitable for physical behaviour of problem, the results of iteraton would be having more accurate. From this point, wit intent of the reducing the computation time, time-wise step-size of the solution procedure is considered as dt=0.01, which is bigger value in the literature for iterative schemes according to best knowledge's of Authors, by using Curve Fitting Initial Guess. Consequence of this approximation the I-DQM provided easily less computational time and acceptable accuracy for BE.

2. DQM AND WEIGHTING COEFFICIENT

DQM was introduced with intent to solve initial and boundary conditions problems by Richard Bellman in 1971 [25]. The calculation of the weighting coefficients are the most important part of the DQM. [25, 26]. Shu and Richards developed a general algebraic method for calculation of weighting coefficient. This method called as Generalized DQM and commonly used by researchers currently [27-31].

The r^{th} order Derivative of a function is given in Eq. 4 according to GDQM. Here, f (x) is a function of x which is described in x∈[a,b]. Besides, $f(x_i)$ shows numerical real values of f(x) and x_i (i = 1,2, ... N) describes value of x in the obtaining domain [31].

$$\frac{d^r f(x_i)}{dx^r} = \sum_{j=1}^{N} a_{ij}^{(r)} f(x_j) \rightarrow i = 1,2,\dots,N \tag{4}$$

The weighting coefficient for r^{th} order derivative describes as $a_{ij}^{(r)}$. Lagrange interpolation function is used in this procedure given in Eq. 5 to 11 [31].

$$l_j(x) = \frac{\phi(x)}{(x-x_j)\phi^{(1)}(x_j)} \rightarrow j = 1,2,\dots,N \tag{5}$$

$$\phi(x) = \prod_{m=1}^{N}(x-x_m)$$
$$\phi^{(1)}(x_j) = \frac{d\phi(x_j)}{dx} = \prod_{m=1,m\neq 1}^{N}(x_j - x_m) \tag{6}$$

$$a_{ij}^{(1)} = \frac{d l_j(x_i)}{dx} = \frac{\phi^{(1)}(x_i)}{(x_i - x_j)\phi^{(1)}(x_j)}$$
$$i,j = 1,2,\dots,N, i \neq j \tag{7}$$

$$a_{ii}^{(1)} = -\sum_{j=1,i\neq j}^{N} a_{ij}^{(1)}, \qquad i = 1,2,\dots,N \tag{8}$$

Similarly,

$$a_{ij}^{(r)} = \frac{d^r l_j(x_i)}{dx^r} = r\left(a_{ii}^{(r-1)} a_{ij}^{(1)} - \frac{a_{ij}^{(r-1)}}{(x_i - x_j)}\right)$$
$$i,j = 1,2,\dots,N, i \neq j, r \geq 2 \tag{9}$$

$$a_{ii}^{(r)} = \frac{d^r l_j(x_i)}{dx^r} = -\sum_{j=1,i\neq j}^{N} a_{ij}^{(r)}, i = 1,2,\dots,N \tag{10}$$

$$a_{ii}^{(r)} = \sum_{k=1}^{N} a_{ik}^{(r-1)} a_{ik}^{(1)}, i,j = 1,2,\dots,N\ ,, r \geq 2 \tag{11}$$

The weighting coefficients are calculated by using Eq. 11 is stated as matrix [$A^{(r)}$] given in Eq.12 [28]

$$\left[A^{(r)}\right]=\left(\frac{d}{dx}\right)^r=\frac{d^r}{dx^r}=\frac{d^{r-1}}{dx^{r-1}}\frac{d}{dx}=\frac{d}{dx}\frac{d^{r-1}}{dx^{r-1}}=\begin{bmatrix} a_{11}^{(r)} & a_{12}^{(r)} & \dots & a_{1N}^{(r)} \\ a_{21}^{(r)} & a_{22}^{(r)} & \dots & a_{2N}^{(r)} \\ \vdots & \vdots & \vdots & \vdots \\ a_{N1}^{(r)} & a_{N2}^{(r)} & \dots & a_{NN}^{(r)} \end{bmatrix} \quad (12)$$

The weighting coefficients given in Eq. 12 is modified as Eq. 13 for second or high order derivatives [31].

$$\left[A^{(r)}\right]=\left[A^{(1)}\right]\left[A^{(r-1)}\right]=\left[A^{(r-1)}\right]\left[A^{(1)}\right] \quad (13)$$

One of the most effective solution technique for GDQM is used the method as iterative. In this study, Newton-Raphson Iteration method (Eq. 14) is combined with GDQM. This approximation called as Iterative Differential Quadrature Method (I-DQM) or Newton-Raphson Diferential Quadrature Method (NR-DQM).

$$x_{i+1}=x_i-\frac{f(x_i)}{f'(x_i)} \quad (14)$$

I-DQM algorithm of BE is given in Eq. 15, 16 and 17. Here, matrix of B is described the “t” wise weighting coefficient and matrix of A is described the “x” wise weighting coefficient in Eq. 15 and 16. Grid points of the weighting coefficients are chosen as uniformly distributed. Grid points of x wise are chosen 6, t wise are chosen 3. Bold “***u***” shows that Jacobian in Newton Raphson Method.

$$f=B^{(1)}\cdot u-\nu\cdot A^{(2)}\cdot u+u\cdot\left(A^{(1)}\cdot u\right) \quad (15)$$

$$f'=B^{(1)}\cdot \boldsymbol{u}-\nu\cdot A^{(2)}\cdot \boldsymbol{u}+\boldsymbol{u}\cdot\left(A^{(1)}\cdot u\right)+u\cdot\left(A^{(1)}\cdot \boldsymbol{u}\right. \quad (16)$$

$$u_{i+1}=u_i-\frac{f}{f'} \quad (17)$$

3. NUMERICAL SOLUTION

Convergence speed and accuracy of iterative methods depends on the initial guess. Usually initial guess of the iterative scheme considers as arbitrary, shown in Equation 18. However, physical behaviours of one or more than one problems were defined by using PDEs, in engineering. So, unlike the previous iterative studies, in this study an initial guess value is used in accordance with the physical nature of the discussed problem. In order to estimate a proper initial curve for given problem an equation settled by using initial and boundary conditions (Eq. 19). The initial guess function given in Equation 19 is used for numerical values of initial guess as seen in Equation 20. This initial guess approximation called as Curve Fitting Initial Guess (CFIG). Thereby, results of numerical solutions are obtained with high accuracy by using dt=0.01. Iteration number is gotten as 7 and solution of BE is performed according I-DQM algorithm.

$$u(x,t)=\begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & \dots & 1 \end{bmatrix} \quad (18)$$

$$g(x,t)=u(x,0)\cdot t \quad (19)$$

$$u(x,t)=\begin{bmatrix} g(x_1,t_1) & g(x_1,t_2) & \dots & g(x_1,t_n) \\ g(x_2,t_1) & g(x_2,t_2) & \dots & g(x_2,t_n) \\ \vdots & \vdots & & \vdots \\ g(x_n,t_1) & g(x_n,t_2) & \dots & g(x_n,t_1) \end{bmatrix} \quad (20)$$

Problem1. In this problem BE is considered with the initial and boundary condition given below in Eq. 21 and 22 [32-37].

$$u(x,0)=\sin(\pi x), \qquad 0\le x\le 1 \quad (21)$$

$$u(0,t)=u(1,t)=0, \qquad t>0 \quad (22)$$

When the initial and boundary conditions of the Problem 1 are examined, it is seen that the function at t = 0 shows sinusoidal behaviour throughout x. The function u(x, t) takes 0 at x = 0 and x = 1 throughout t. Taking these two conditions into account, the initial guess of the function u(x, t) is evaluated as a sine function of x. That is, a curve fitting is provided for the required initial guess or CFIG in the iteration procedure.

Analytic solution of Problem 1 is obtained by using Hopf-Cole transformation (Eq. 23, 24 and 25).

$$u(x,t)=\frac{2\pi\nu\sum_{n=1}^{\infty}B_n\exp(-n^2\pi^2\nu\, t)\, n\, sin(n\pi x)}{B_0+\sum_{n=1}^{\infty}B_n\exp(-n^2\pi^2\nu\, t)\, cos(n\pi x)} \quad (23)$$

$$B_0=\int_0^1\exp\left(\frac{-1}{2\pi\nu}(1-cos\,(\pi x)\right)dx \quad (24)$$

$$B_n=2\int_0^1\exp\,\left(\frac{-1}{2\pi\nu}(1-\cos\,(\pi x)\right)\cos\,(n\pi x)dx \quad (25)$$

Comparison of I-DQM results with pervious numerical studies are given in Table 1 to 6 respectively for Problem 1. When the tables are examined in detail, the average absolute error of I-DQM solutions are obtained 1.04×10^{-4}. Therefore, I-DQM is ensured very high accuracy for solution BE. Besides, step size of time wise is used as dt=0.01 in the proposed method. Thus, an accurate solution is reached in a short time.

Numerical solutions of first problem of the BE by Using I-DQM with presented initial guess procedure are given in Fig. 1 and Fig. 2 as graphical for ν=0.001 and ν=0.0001, respectively. The smaller viscosity value causes the more nonlinearity for the given problem. Thereby, Fig. 1 and Fig. 2 shows that proposed initial guess procedure provide reliable solution for highly nonlinear problems by using I-DQM

Table 1. Solution of Problem 1 by using I-DQM v=0.01, dt=0.01

x	t	u(x,t) Hopf-Cole	Present Results dt=0.01	Ref 18 dt= 0.001	Ref 17 dt=0.001
0.25	0.4	0.308894	0.308845	0.30887	0.30889
	0.6	0.240739	0.240709	0.2407	0.24075
	0.8	0.195676	0.195656	0.19566	0.19569
	1	0.162565	0.162552	0.16255	0.16258
	3	0.027202	0.027200	0.02721	0.0272
0.5	0.4	0.569632	0.569609	0.56956	0.56963
	0.6	0.447206	0.447180	0.44715	0.44724
	0.8	0.359236	0.359215	0.3592	0.35927
	1	0.291916	0.291899	0.29188	0.29195
	3	0.040205	0.040201	0.04022	0.04021
0.75	0.4	0.625438	0.625525	0.6254	0.62537
	0.6	0.487215	0.487247	0.48716	0.48718
	0.8	0.373922	0.373925	0.37389	0.37391
	1	0.287474	0.287466	0.28743	0.28747
	3	0.029772	0.029769	0.02978	0.02977

Table 1. Solution of Problem 1 by using I-DQM v=0.01, dt=0.01

x	t	u(x,t) Hopf-Cole	Present Results dt=0.01	Ref 19 dt=0.001	Ref 18 dt= 0.001	Ref 17 dt=0.001
0.25	0.4	0.341915	0.341856	0.341915	0.34184	0.34191
	0.6	0.268965	0.268917	0.268966	0.26891	0.26896
	0.8	0.221482	0.221445	0.221482	0.22143	0.22148
	1	0.188194	0.188165	0.188193	0.18815	0.18820
	3	0.075114	0.075108	0.075114	0.07510	0.07511
0.5	0.4	0.660711	0.660715	0.660713	0.66060	0.66069
	0.6	0.529418	0.529377	0.529419	0.52932	0.52942
	0.8	0.439138	0.439090	0.439138	0.43905	0.43914
	1	0.374420	0.374376	0.374420	0.37436	0.37443
	3	0.150179	0.150168	0.150179	0.15017	0.15019
0.75	0.4	0.910260	0.910508	0.910250	0.91026	0.91023
	0.6	0.767240	0.767324	0.767230	0.76719	0.76723
	0.8	0.647400	0.647394	0.647400	0.64745	0.64740
	1	0.556050	0.556022	0.556048	0.55608	0.55606
	3	0.224811	0.224796	0.224812	0.22504	0.22486

Table 3. Solution of Problem 1 by using I-DQM v=0.005, dt=0.01

x	t	u(x,t) Hopf-Cole	Present Results dt=0.01	Ref 19 dt=0.001	Ref 18 dt= 0.001	Ref 17 dt=0.001
0.25	1	0.188788	0.188758	0.188787	0.18874	0.18898
	5	0.046963	0.046961	0.046962	0.04695	0.04698
	10	0.024217	0.024216	0.024217	0.02421	0.02422
	15	0.016308	0.016307	0.016305	0.01631	0.01631
0.5	1	0.375723	0.375677	0.375724	0.37565	0.37608
	5	0.093920	0.093915	0.093921	0.09391	0.09396
	10	0.048421	0.048420	0.048424	0.04842	0.04843
	15	0.032439	0.032438	0.032439	0.03244	0.3244
0.75	1	0.558380	0.558352	0.55838	0.55831	0.55883
	5	0.140832	0.140824	0.140836	0.14083	0.14091
	10	0.071134	0.071132	0.071134	0.07114	0.07118
	15	0.044133	0.044132	0.044135	0.04415	0.04416

Table 2 Solution of Problem 1 by using I-DQM v=0.004, dt=0.01

x	t	u(x,t) Hopf-Cole	Present Results dt=0.01	Ref 18 dt= 0.001	Ref 17 dt=0.001
0.25	1	0.188904	0.188857	0.18886	0.18891
	5	0.046972	0.046970	0.04696	0.04697
	10	0.024219	0.024219	0.02421	0.02422
	15	0.016315	0.016315	0.01631	0.01632
0.5	1	0.375960	0.375921	0.37591	0.37598
	5	0.093938	0.093933	0.09393	0.09394
	10	0.048437	0.048436	0.04843	0.04843
	15	0.032595	0.032594	0.03259	0.03259
0.75	1	0.558810	0.558797	0.55875	0.55883
	5	0.140887	0.140880	0.14088	0.14089
	10	0.072202	0.072200	0.07221	0.07221
	15	0.046775	0.046774	0.04679	0.04678

Table 5 Solution of Problem 1 by using I-DQM v=0.003, dt=0.01

x	t	u(x,t) Hopf-Cole	Present Results dt=0.01	Ref 19 dt=0.001.	Ref 18 dt= 0.001	Ref 17 dt=0.001
0.25	1	0.189019	0.188823	0.18902	0.18898	0.18902
	5	0.046981	0.046978	0.04698	0.04697	0.04698
	10	0.024222	0.024221	0.02422	0.02422	0.02422
	15	0.016317	0.016317	0.01632	0.01631	0.01631
0.5	1	0.376190	0.376073	0.37616	0.37615	0.37623
	5	0.093955	0.093950	0.09396	0.09394	0.09396
	10	0.048430	0.048442	0.04842	0.04843	0.04844
	15	0.032632	0.032631	0.03263	0.03263	0.03263
0.75	1	0.559240	0.559180	0.55927	0.55919	0.55928
	5	0.140950	0.140909	0.14094	0.14091	0.14092
	10	0.072600	0.072601	0.07259	0.07261	0.07261
	15	0.048410	0.048385	0.04840	0.04840	0.04839

Table 6. Solution of Problem 1 by using I-DQM dt=0.01

		v=0.1		v=0.01		v=0.005	
x	**t**	**u(x,t) Hopf-Cole**	**Present Results dt=0.01**	**u(x,t) Hopf-Cole**	**Present Results dt=0.01**	**u(x,t) Hopf-Cole**	**Present Results dt=0.01**
0.8	0.4	0.571267	0.571368	0.94103	0.941323	0.95230	0.953291
	0.6	0.442263	0.442305	0.80955	0.809677	0.81411	0.815979
	0.8	0.334680	0.334687	0.68708	0.687103	0.69095	0.690947
	1	0.253718	0.253712	0.59148	0.591456	0.59409	0.594069
	3	0.024918	0.024915	0.23863	0.238617	0.24050	0.240479
0.9	0.4	0.350951	0.351037	0.95244	0.952658	0.97814	0.978999
	0.6	0.267802	0.267840	0.88422	0.884466	0.89395	0.894342
	0.8	0.197394	0.197402	0.76295	0.763031	0.76814	0.768617
	1	0.146065	0.146062	0.66002	0.660026	0.66405	0.664343
	3	0.013226	0.013225	0.24158	0.241564	0.26901	0.268993

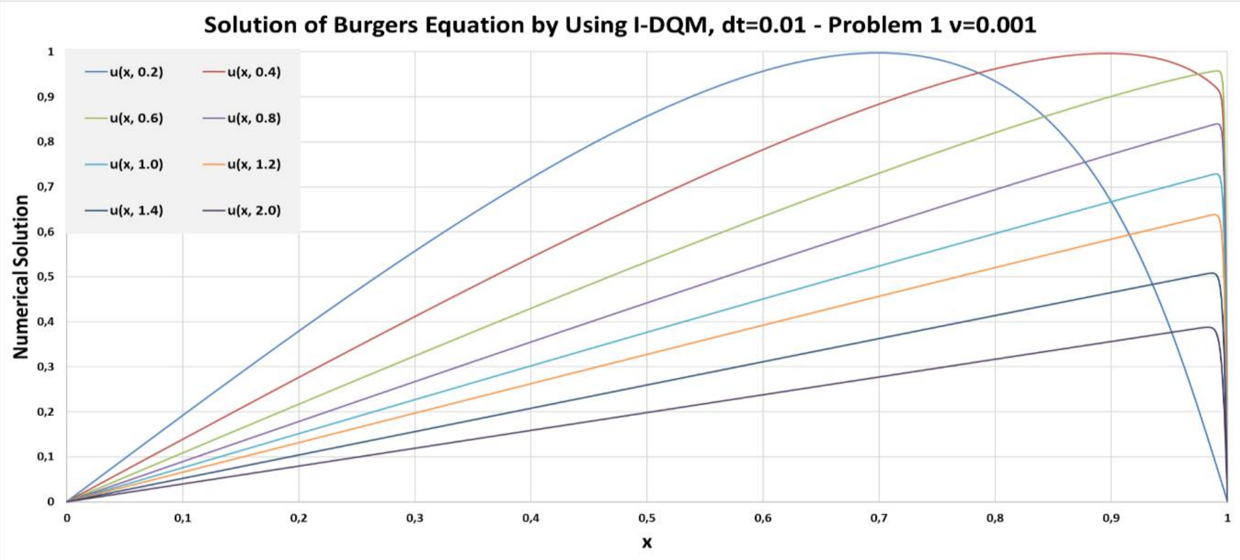

Figure 2 Numerical Solution of Burgers Equation by Using I-DQM, dt=0.01 - Problem 1 ν=0.001

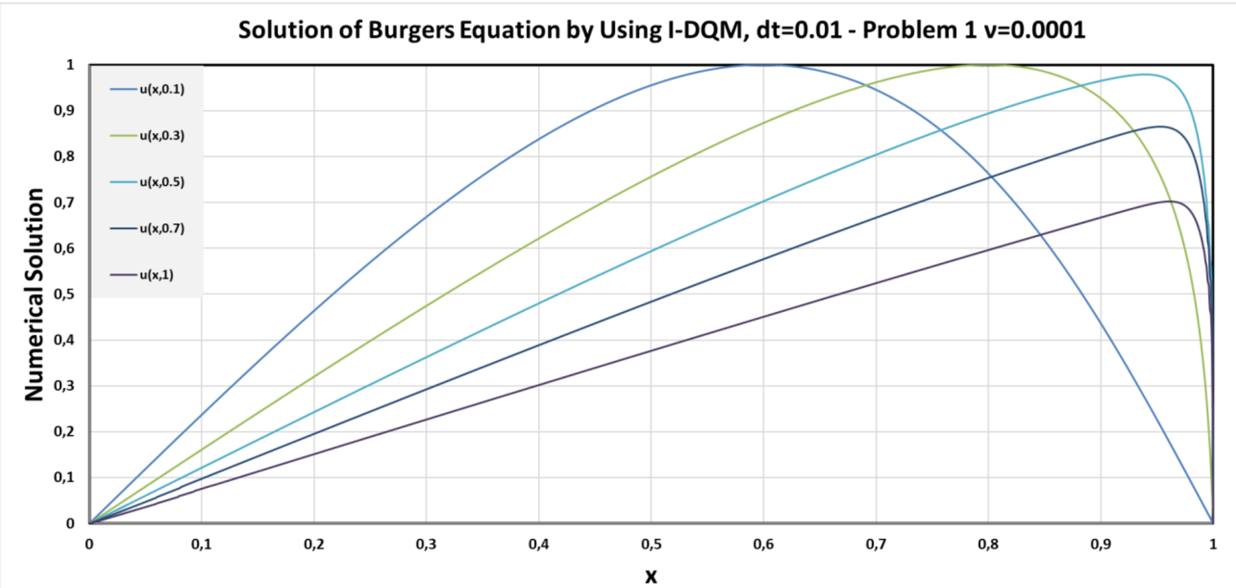

Figure 1 Numerical Solution of Burgers Equation by Using I-DQM, dt=0.01 - Problem 1 ν=0.0001

Table 7. Solution of Problem 2 by using I-DQM v=0.1, dt=0.01

x	t	u(x,t) Hopf-Cole	Present Results dt=0.01	Ref 18 dt= 0.001	Ref 17 dt=0.001
0.25	0.4	0.317523	0.317469	0.30887	0.31752
	0.6	0.246138	0.246107	0.24609	0.24615
	0.8	0.199555	0.199535	0.19952	0.19957
	1	0.165599	0.165585	0.16557	0.16561
	3	0.027759	0.027756	0.02775	0.02776
0.5	0.4	0.584537	0.584522	0.56979	0.58454
	0.6	0.457976	0.457953	0.4579	0.458
	0.8	0.367398	0.367378	0.36734	0.36744
	1	0.298343	0.298326	0.29829	0.29838
	3	0.041065	0.041061	0.07105	0.04107
0.75	0.4	0.645616	0.645706	0.62567	0.64556
	0.6	0.502676	0.502712	0.48715	0.50265
	0.8	0.385336	0.385341	0.38525	0.38532
	1	0.295857	0.295849	0.29578	0.29585
	3	0.030440	0.030437	0.03043	0.03044

Table 8. Solution of Problem 2 by using I-DQM v=0.01, dt=0.01.

x	t	u(x,t) Hopf-Cole	Present Results dt=0.01	Ref 19 dt=0.001	Ref 18 dt= 0.001	Ref 17 dt=0.001
0.25	0.4	0.362259	0.362199	0.362258	0.36217	0.36225
	0.6	0.282037	0.281984	0.282034	0.28197	0.28204
	0.8	0.230451	0.230409	0.230453	0.2304	0.23045
	1	0.194690	0.194658	0.194694	0.19645	0.19649
	3	0.076134	0.076128	0.076139	0.07613	0.07613
0.5	0.4	0.683679	0.683707	0.683672	0.68357	0.68364
	0.6	0.548316	0.548289	0.548312	0.54822	0.54831
	0.8	0.453714	0.453671	0.453715	0.45363	0.45371
	1	0.385676	0.385633	0.385671	0.38561	0.38568
	3	0.152180	0.152168	0.152173	0.15217	0.15219
0.75	0.4	0.920500	0.920704	0.920510	0.9205	0.92044
	0.6	0.782990	0.783085	0.782990	0.78293	0.78297
	0.8	0.662707	0.662734	0.662703	0.66264	0.66272
	1	0.569321	0.569300	0.569321	0.56924	0.56932
	3	0.227743	0.227727	0.227745	0.22774	0.22779

Problem 2 Secondly, the BE is considered with the initial and boundary conditions given below Eq 26 and 27 [32-37]. The CFIG approach is used for Problem 2 similar to Problem 1.

$$(x,0) = 4\text{x}(1-\text{x}), \qquad 0 \le x \le 1 \tag{26}$$

$$u(0,t) = u(1,t) = 0, \qquad t > 0 \tag{27}$$

Analytic solution of Problem 2 is calculated by using Hopf-Cole transformation (Eq. 28, 29 and 30).

$$(x,t) = \frac{2\pi\nu \sum_{n=1}^{\infty} B_n \exp(-n^2\pi^2\nu\, t)\, n\, sin(n\pi x)}{B_0 + \sum_{n=1}^{\infty} B_n \exp(-n^2\pi^2\nu\, t)\, cos(n\pi x)} \tag{28}$$

$$B_0 = \int_0^1 \exp\left(\frac{-1}{3\nu}(3x^2 - 2x^3)\right) dx \tag{29}$$

$$B_n = 2\int_0^1 \exp\left(\frac{-1}{3\nu}(3x^2 - 2x^3)\right) cos(n\pi x)\, dx \tag{30}$$

Comparison of I-DQM results with pervious numerical studies in the literature are given in Table 7 to 12 respectively for Problem 2. Mean absolute error of the I-DQM solutions are obtained as $9.3\text{x}10^{-5}$ by using step size in time wise as dt=0.01.

Table 9. Solution of Problem 2 by using I-DQM v=0.005, dt=0.01.

x	t	u(x,t) Hopf-Cole	Present Results dt=0.01	Ref 19 dt=0.001	Ref 18 dt= 0.001	Ref 17 dt=0.001
0.25	1	0.196081	0.196046	0.196081	0.19604	0.19630
	5	0.047415	0.047412	0.047415	0.04741	0.04740
	10	0.024336	0.024335	0.024335	0.02433	0.02434
	15	0.016362	0.016361	0.016362	0.01636	0.01636
0.5	1	0.387970	0.387971	0.387950	0.38795	0.38839
	5	0.094814	0.094809	0.094812	0.09481	0.09487
	10	0.048660	0.048658	0.048655	0.04866	0.04867
	15	0.032550	0.032549	0.032552	0.03255	0.03256
0.75	1	0.572500	0.572542	0.572500	0.57248	0.57299
	5	0.142154	0.142146	0.142155	0.14215	0.14224
	10	0.071517	0.071515	0.071519	0.07152	0.07157
	15	0.044328	0.044327	0.044339	0.04433	0.04436

Table10. Solution of Problem 2 by using I-DQM v=0.004, dt=0.01

x	t	u(x,t) Hopf-Cole	Present Results dt=0.01	Ref 18 dt= 0.001	Ref 17 dt=0.001
0.25	1	0.196393	0.196424	0.19636	0.19400
	5	0.047439	0.047440	0.04744	0.04744
	10	0.024343	0.024342	0.02434	0.02434
	15	0.016371	0.016371	0.01637	0.01637
0.5	1	0.388460	0.388550	0.38842	0.38850
	5	0.949300	0.094864	0.09491	0.09487
	10	0.048683	0.048682	0.04868	0.04868
	15	0.032707	0.032706	0.03270	0.03271
0.75	1	0.573150	0.573280	0.57312	0.57320
	5	0.142248	0.142254	0.14224	0.14225
	10	0.072581	0.072579	0.07258	0.07258
	15	0.046964	0.046963	0.04696	0.04696

Table11. Solution of Problem 2 by using I-DQM v=0.003, dt=0.01.

x	t	u(x,t) Hopf-Cole	Present Results dt=0.01	Ref 19. dt=0.001	Ref 18 dt= 0.001	Ref 17 dt=0.001
0.25	1	0.196722	0.196753	0.196725	0.19668	0.19673
	5	0.047465	0.047467	0.047468	0.04746	0.04747
	10	0.024350	0.024350	0.024351	0.02434	0.02434
	15	0.016375	0.016375	0.016379	0.01637	0.01637
0.5	1	0.389246	0.389026	0.38925	0.3889	0.38898
	5	0.094912	0.094915	0.094923	0.09491	0.09491
	10	0.048698	0.048699	0.048695	0.0487	0.04869
	15	0.032748	0.032748	0.032749	0.03274	0.03275
0.75	1	0.573780	0.573911	0.87376	0.57375	0.57383
	5	0.142324	0.142329	0.142326	0.14232	0.14233
	10	0.072986	0.072987	0.072982	0.07298	0.07299
	15	0.048568	0.048569	0.048565	0.04857	0.04857

Table 3 Solution of Problem 2 by using I-DQM dt=0.01.

		v=0.1		v=0.01		v=0.005	
x	**t**	**u(x,t) Hopf-Cole**	**Present Results dt=0.01**	**u(x,t) Hopf-Cole**	**Present Results dt=0.01**	**u(x,t) Hopf-Cole**	**Present Results dt=0.01**
0.8	0.4	0.591355	0.591457	0.948630	0.948861	0.958490	0.959473
	0.6	0.457392	0.457438	0.823720	0.823849	0.830150	0.830520
	0.8	0.345593	0.345603	0.701950	0.701987	0.766120	0.706611
	1	0.261539	0.261534	0.604780	0.604772	0.608260	0.608253
	3	0.025480	0.025478	0.241798	0.241781	0.243951	0.243933
0.9	0.4	0.365460	0.365543	0.962990	0.963180	0.982340	0.983273
	0.6	0.278234	0.278276	0.894770	0.894983	0.903980	0.904464
	0.8	0.204559	0.204569	0.776380	0.776465	0.783480	0.782567
	1	0.150974	0.150972	0.673114	0.673141	0.676150	0.678119
	3	0.013528	0.013527	0.245633	0.245612	0.272899	0.272894

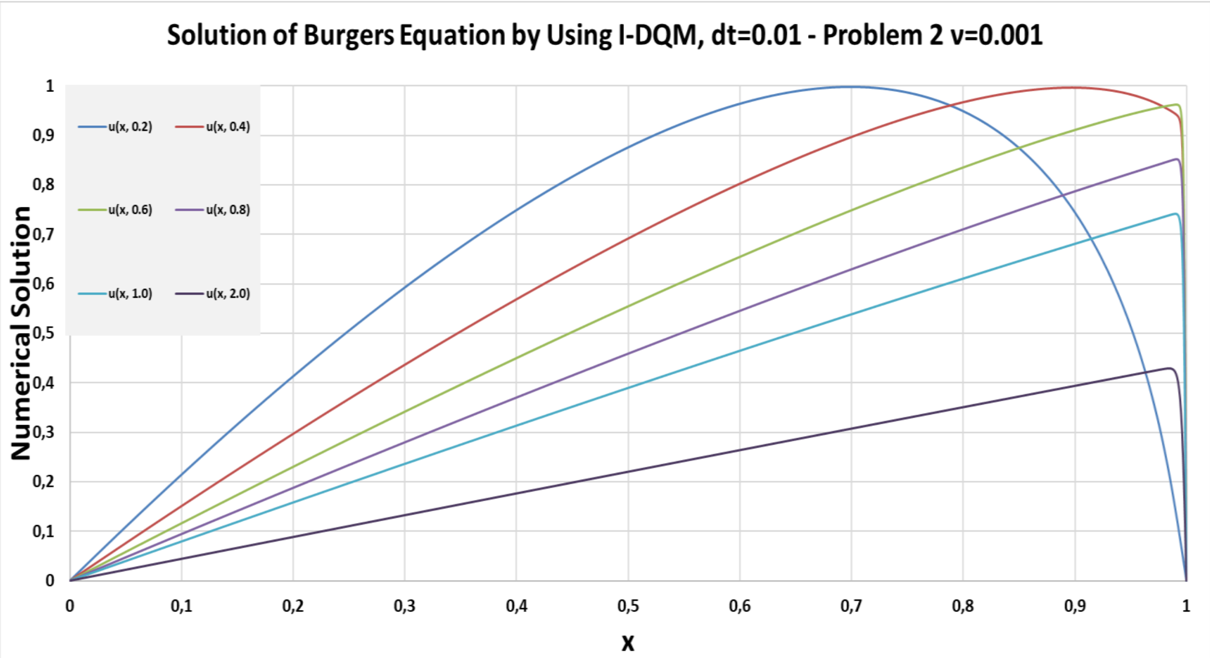

Figure 3 Numerical Solution of Burgers Equation by Using I-DQM, dt=0.01 - Problem 2 ν=0.001

Numerical solutions of the problem 2 by Using I-DQM with curve fitting initial guess method are given in Fig. 3 and Fig. 4 as graphical for ν=0.001 and ν=0.0001, respectively. Cause of smaller viscosity value makes given problem more nonlinear, the solution of BE became harder for small viscosities. However, Fig. 3 and Fig. 4 shows that proposed iterative scheme provide reliable solution for very small kinematic viscosity values. Thus, curve fitting initial guess method ensured reliable solution for highly nonlinear problems by using I-DQM.

$$e = \frac{|u_{real} - u_{numerical}|}{u_{real}} \qquad (31)$$

It is seen from the given tables and figures that the results obtained with the curve fitting initial guess method have a very high accuracy. The error analysis of the results performed in order to clarify the accuracy of the solution. Absolute error is calculated by using Equation 31, in order to make error analysis easier and understandable. Mean Relative Error (MRE) value is obtained by calculating the mean of absolute errors of all numerical results. The MRE of I-DQM solutions of Problem 1 obtained as $1.04\text{x}10^{-4}$. However, MRE of Ref. 17 and Ref. 18 for Problem 1 calculated as $1.74\text{x}10^{-4}$ and $1.55\text{x}10^{-4}$, respectively. MRE of the I-DQM solutions of Problem 2 are obtained as $9.3\text{x}10^{-5}$. Also MRE of Ref. 17 and Ref. 18 of Problem 2 calculated $1.31\text{x}10^{-2}$ and

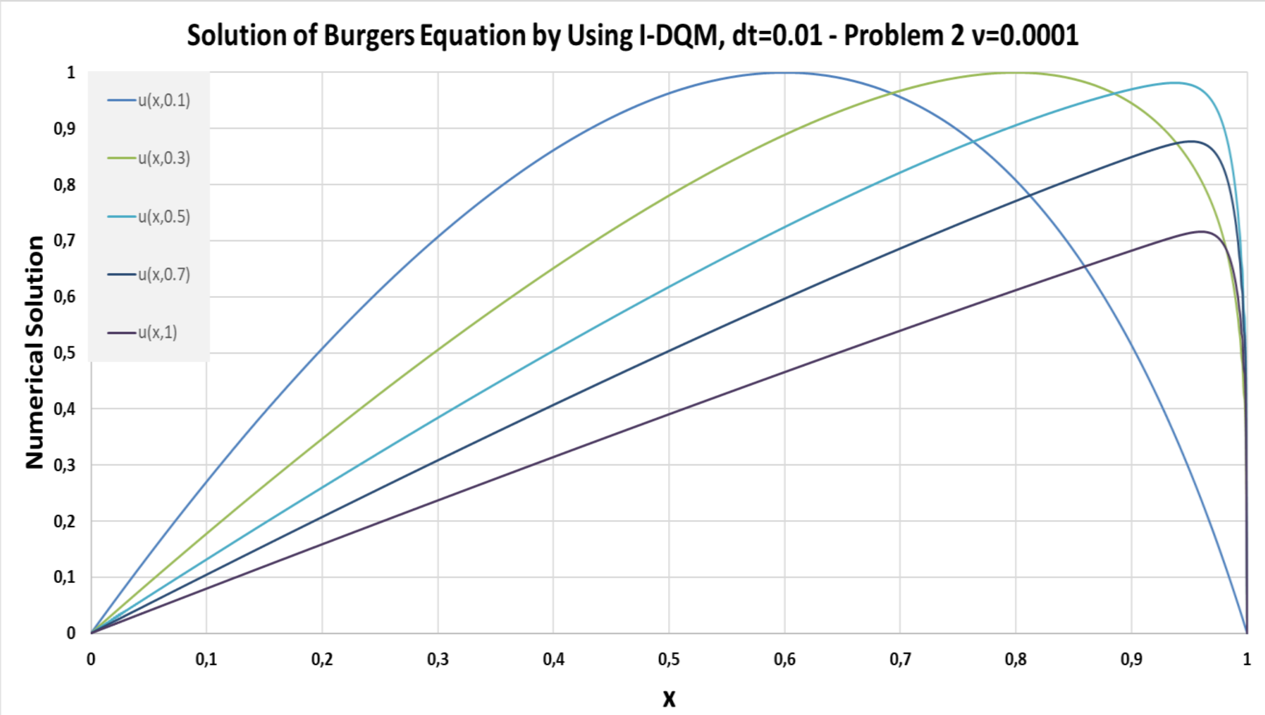

Figure 4 Numerical Solution of Burgers Equation by Using I-DQM, dt=0.01 - Problem 2 ν=0.001

$5.02\text{x}10^{-4}$. As a result of the error analysis, it is observed that the accuracy of the solution is obtained 10 times better than other studies, especially in Problem 2. Therefore, I-DQM is ensured very high accuracy for solution BE. Besides, step size of time wise is used as dt=0.01 in the proposed method. Thus, an accurate solution is reached in a short time. The results of both Problem 1 and Problem 2 are indicated that the numerical solution of BE by using I-DQM is ensured that sensitively 10 times faster solution than other iterative studies in the literature. So, the results of the error analysis of the I-DQM algorithm with CFIG is shown that the proposed method ensured the stability for the numerical solution of BE.

6. CONCLUSION

In this study, numerical solution of BE performed with I-DQM as taking dt=0.01. Thus, a significant gain achieved in the calculation time. This situation is provided by using Curve Fitting Initial Guess (CFIG) approach. The error analysis of obtaining results compared with the previous works and significantly high accuracy ensured even though dt=0.01. The smaller viscosity value causes the more nonlinearity for the observed problem. Furthermore, numerical solutions of the problem 1 and 2 by using I-DQM with CFIG method are provided even for ν=0.001 and ν=0.0001. Consequently, these results prove that the numerical solution of BE could be calculated easily with very high accuracy in a short time by using I-DQM with CFIG approach.

ACKNOWLEDGEMENT

This study is supported by Afyon Kocatepe University Scientific Research Projects Commission with 17.FEN.BIL.76 numbered project.

DECLARATION OF ETHICAL STANDARDS

The author(s) of this article declare that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission..

AUTHORS' CONTRIBUTIONS

Zekeriya GİRGİN: Developed and Perofrmed the numerical shceme and analyse the results.

Faruk Emre AYSAL: Developed and Perofrmed the numerical shceme and analyse the results. Wrote the manuscript.

Hüseyin BAYRAKÇEKEN: Analyzed the numerical scheme results in terms of absolute error.

CONFLICT OF INTEREST

There is no conflict of interest in this study.

REFERENCES

[1]. Bateman H., "Some Recent Researches on The Motion of Fluids", ***Monthly Weather Review***, 40: 163-170 (1915).

[2]. Burgers J. M., "Mathematical examples illustrating relations occurring in the theory of turbulent fluid motion", ***Trans. Roy. Neth. Acad. Sci.,*** 17 (2): 1-5 (1939).

[3]. Burgers J. M., "A Mathematical Model Illustrating the Theory of Turbulence", ***Advances in Applied Mechanics***, 1: 171-199 (1948).

[4]. Hopf E., "The partial differential equation ut + uux = μxx", ***Pure and Applied Mathematics***, 3(3): 201-230 (1950).

[5]. Cole J. D., "On a quasi-linear parabolic equation occurring in aerodynamics", ***Quart. Appl. Math.,*** 9: 225-236 (1951).

[6]. Caldwell J. and Smith P., "Solution of Burgers' equation with a large Reynolds number", ***Appl. Math. Model.*** 6: 381-385 (1982).

[7]. Evans D. J. and Abdullah A. R., "The group explicit method for the solution of Burgers' equation", ***Quart. Appl. Math***., 30: 239-253 (1984).

[8]. Mittal R. C. and Signnal P., "Numerical solution of Burgers' equation", ***Commun. Numer. Methods Eng,*** 9: 397-406 (1993).

[9]. Öziş T. and Özdeş A., "A direct variational methods applied to Burgers' equation", ***J. Comput. Appl. Math***, 71: 163-175 (1996).

[10]. Kutluay S. A., Bahadir A. R. and Özdeş A., "Numerical solution of one-dimensional Burgers equation: explicit and exact-explicit finite difference methods", ***J. Comput. Appl. Math***, 103: 251-261 (1999).

[11]. Kutluay S. A. and Esen A., "A linearized numerical scheme for Burgers-like equations", ***Appl. Math. Comput***, 156**:** 295-305 (2004).

[12]. Liao W., "An implicit fourth-order compact finite difference scheme for one-dimensional Bugers equation", ***Appl. Math. Comput***, 206: 755-764 (2008).

[13]. Öziş T. and Erdoğan U., "An exponentially fitted method for solving Bugers equation", ***Int. J. Numer. Meth. Engng***. 79: 696-705 (2009).

[14]. Gao Q. and Zou M.Y., "An analytical solution for two and three dimensional nonlinear Burgers' equation", ***Applied Mathematical Modelling***. 45: 255–270 (2017).

[15]. Mittal R. and Jiwari R., "A differential quadrature method for numerical solutions of Burgers'-type equations", ***International Journal of Numerical Methods for Heat & Fluid Flow***, 22(7): 880-895 (2012).

[16]. Gupta S. ad Ray S., "Comparison between homotopy perturbation method and optimal homotopy asymptotic method for the soliton solutions of Boussinesq–Burger equations", ***Computers and Fluids***, 103: 34-41 (2014).

[17]. Nascimento A., Silveria-Neto F. P. M. A. and Padilla E. L. M., "A comparison of Fourier pseudospectral method and finite volume method used to sol, the Burgers equation", ***J Braz. Soc. Mech. Sci. Eng***. 36: 737-742 (2014).

[18]. Jiwari R., "A hybrid numerical scheme for the numerical solution of the Burgers' equation", ***Computer Physics Communications,*** 188: 59-67 (2015).

[19]. Jiwari R., "A Haar wavelet quasilinearization approach for numerical simulation of Bugers equation", ***Computer Physics Communications,*** 193: 2413-2423 (2012).

[20]. Tamsir M., Srivastava V. K. and Jiwari R., "An algorithm based on exponential modified cubic B-spline differential quadrature method for nonlinear Burgers' equation", ***Applied Mathematics and Computation***. 290 111-124 (2016).

[21]. Girgin Z., Aysal F. E., Bayrakçeken H., "Numerical Solution of the Burgers Equation by Using Iterative DQM", ***5th International Symposium on Innovative Technologies in Engineering and Science***, 268-277 (2017).

[22]. Civalek Ö, Kiracioglu O, "Free vibration analysis of Timoshenko beams by DSC method", **Int J Numer Methods Biomed Eng,** 26(12): 1890–1898 (2010).

[23]. Civalek, Ö. and Yavas A. "Large deflection static analysis of rectangular plates on two parameter elastic foundations", ***International Journal of Science and Technology,*** 1(1): 43–50 (2006).

[24]. Mercan K, Demir Ç. and Civalek Ö., "Vibration analysis of FG cylindrical shells with power-law index using discrete singular convolution technique". ***Curved Layer Struct,*** 3(1**):** 82-90 (2016).

[25]. Bellman R. and Casti J., "Differential quadrature and long-term integration", ***J. Math. Anal. Appl.,*** 34(2): 235-238 (1971).

[26]. Bellman R., Kashef B. G. J. "Casti, Differential quadrature: A technique for the rapid solution of nonlinear partial differential equations", ***Journal of Computational Physics***, 40(1): 40-52 (1972).

[27]. Bellman R., Kashef B. G, Lee E. and Vasudevan S., R., "Differential quadrature and splines", ***Computers and Mathematics with Applications***. 1(3-4): 371-376 (1975)

[28]. Quan J. R. and Chang C. T., "New Insights in Solving Distributed System Equations by The Quadrature Methods – I", ***Computational Chemical Engineering***. 13: 779-788 (1989).

[29]. Quan J. R. and Chang C. T., "New Insights in Solving Distributed System Equations by The Quadrature Methods – II", ***Computational Chemical Engineering***. 13: 1017-1024 (1989).

[30]. Shu B. and Richards E., "High Resolution of Natural Convection in A Square Cavity by Generalized Differential Quadrature", ***Proceeding of 3rd Conference on Advanced in Numerical Methods in Engineering: Theory and Application, Swansea, UK***, 2: 978-985 (1990).

[31]. Shu B., "Generalized Differential-Integral Quadrature and Application to The Simulation of Incompressible Viscous Flows Including Paralel Computation", ***PhD Dissertation. Uni,rsity of Glosgow, UK.,*** (1991).

[32]. Doğan A., "A Galerkin finite element approach to Burgers' equation", ***Appl. Math. Comput.*** 157: 331-346 (2004).

[33]. Kadalbajoo M. and Awasthi A., "A numerical method based on Crank–Nicolson scheme for Burgers' equation", ***Appl. Math. Comput***, 182: 1430-1442 (2006).

[34]. Xu M., Wang R.-H., Zhang J.-H. and Fang Q., "A novel numerical scheme for solving Burgers' equation", ***Appl. Math. Comput***. 217: 4473-4482 (2011).

[35]. Başhan A., "A numerical treatment of the coupled viscous Burgers' equation in the presence of very large Reynolds number" ***Physica A: Statistical Mechanics and its Applications.*** 545: 123755 (2020).

[36]. Ucar Y., Yağmurlu N. M. and Başhan A. "Numerical Solutions and Stability Analysis of Modified Burgers Equation via Modified Cubic B-Spline Differential Quadrature Methods." ***Sigma: Journal of Engineering & Natural Sciences.*** 37 (1): 129-142 (2019).

[37]. Başhan, A,. Karakoç S. B. G and Geyikli T. "B-spline differential quadrature method for the modified Burgers' equation." ***Çankaya Üniversitesi Bilim ve Mühendislik Dergisi.*** 12(1): 001–013 (2015).